Déterminer les primitives des fonctions suivantes sur l'intervalle $]0\,;\frac{\pi}{2}[$.

a) $f(x) = \dfrac{\sin x}{\cos x}$.

$u = \cos x \Rightarrow u' = -\sin x$.

$f(x) = \dfrac{\sin x}{\cos x} = -\left(\dfrac{-\sin x}{\cos x}\right) = -\dfrac{u'}{u} \Rightarrow F(x) = -\ln|u| + k = -\ln|\cos x| + k = -\ln(\cos x) + k$, car $0 < \cos x < 1$ sur $]0\,;\frac{\pi}{2}[$.

b) $g(x) = \dfrac{\sin^3 x}{\cos x}$.

$g(x) = \dfrac{\sin^3 x}{\cos x} = \dfrac{\sin x\,(1-\cos^2 x)}{\cos x} = \dfrac{\sin x}{\cos x} - \sin x.\cos x$.

$u = \cos x \Rightarrow u' = -\sin x$, donc $\sin x.\cos x = -u.u'$ de primitive $\dfrac{u^2}{2}$.

En utilisant a) on déduit :

$g(x) = \dfrac{\sin^3 x}{\cos x} = \dfrac{\sin x}{\cos x} - \sin x.\cos x \Rightarrow G(x) = -\ln(\cos x) + \dfrac{1}{2}\cos^2 x + k$.

c) $h(x) = \dfrac{\sin x}{\cos^2 x}$.

$h(x) = \dfrac{\sin x}{\cos^2 x} = -\left(-\dfrac{u'}{u^2}\right) \Rightarrow H(x) = \dfrac{1}{u} + k = \dfrac{1}{\cos x} + k$.