Calculer les dérivées des fonctions suivantes :

a) $f(x) = \sqrt{x^2 + x + 1}$ **.**

$f = \sqrt{u} \Rightarrow f' = \dfrac{u'}{2\sqrt{u}}$.

$f'(x) = \dfrac{(x^2 + x + 1)'}{2\sqrt{x^2 + x + 1}} = \dfrac{2x + 1}{2\sqrt{x^2 + x + 1}}$.

b) $g(x) = \sqrt{\dfrac{x + 1}{x - 1}}$ **.**

$g = \sqrt{U} \Rightarrow g' = \dfrac{U'}{2\sqrt{U}}$.

$U = \dfrac{u}{v} \Rightarrow U' = \dfrac{u'v - v'u}{v^2}$.

$$g'(x) = \dfrac{\left(\dfrac{x + 1}{x - 1}\right)'}{2\sqrt{\dfrac{x + 1}{x - 1}}} = \dfrac{\dfrac{1.(x - 1) - 1.(x + 1)}{(x - 1)^2}}{2\sqrt{\dfrac{x + 1}{x - 1}}} = \dfrac{\dfrac{-2}{(x - 1)^2}}{2\sqrt{\dfrac{x + 1}{x - 1}}} = -\dfrac{1}{(x - 1)^2}\sqrt{\dfrac{x - 1}{x + 1}} .$$

On constate que $g'(x) < 0$, pour tout x de son domaine de définition.

c) $h(x) = \dfrac{1}{\sqrt{x^2 + 1}}$ **.**

$h = \dfrac{1}{U} \Rightarrow h' = -\dfrac{U'}{U^2}$,

$U = \sqrt{u} \Rightarrow U' = \dfrac{u'}{2\sqrt{u}}$.

$$h'(x) = -\dfrac{(\sqrt{x^2 + 1})'}{x^2 + 1} = -\dfrac{\dfrac{2x}{2\sqrt{x^2 + 1}}}{x^2 + 1} ,$$

$$h'(x) = -\dfrac{x}{(x^2 + 1)\sqrt{x^2 + 1}} .$$