

Calculer les primitives de :

a) $f(x) = \frac{4x-2}{\sqrt{x^2-x-1}}$ sur $I = [2; +\infty[$.

$$f = \sqrt{u} \Rightarrow f' = \frac{u'}{2\sqrt{u}}, \text{ avec } u = x^2 - x - 1 \Rightarrow u' = 2x - 1.$$

$$f(x) = \frac{4x-2}{\sqrt{x^2-x-1}} = 4 \left(\frac{2x-1}{2\sqrt{x^2-x-1}} \right) = 4 \left(\frac{u'}{2\sqrt{u}} \right) \Rightarrow F_k(x) = 4\sqrt{u} + k = 4\sqrt{x^2-x-1} + k, \forall k \in \mathbb{R}.$$

b) $f(x) = \frac{2}{\sqrt{1-2x}} + \frac{x}{\sqrt{x^2+2}}$ sur $I =]-\infty; 0[$.

$$u = 1 - 2x \Rightarrow u' = -2 \text{ et } v = x^2 + 2 \Rightarrow v' = 2x.$$

$$f(x) = \frac{2}{\sqrt{1-2x}} + \frac{x}{\sqrt{x^2+2}} = -2 \left(\frac{-2}{2\sqrt{1-2x}} \right) + \frac{2x}{2\sqrt{x^2+2}} = -2 \left(\frac{u'}{2\sqrt{u}} \right) + \frac{v'}{2\sqrt{v}},$$

$$\text{D'où : } F_k(x) = -2\sqrt{u} + \sqrt{v} + k = -2\sqrt{1-2x} + \sqrt{x^2+2} + k, \forall k \in \mathbb{R}.$$