**Soit** $\boldsymbol{S_{n\,,\,p} = \sum_{k=1}^{n} \frac{k^p}{2^k}}$ **.**

**Calculer** $\boldsymbol{S_{n\,,\,0}\ ,\ S_{n\,,\,1}\ ,\ S_{n\,,\,2}}$ **.**

$S_{n\,,\,0} = \sum_{k=1}^{n} \frac{1}{2^k}$ , somme des termes d'une suite géométrique de $n$ termes, de 1er terme $\frac{1}{2}$ , de raison $q = \frac{1}{2}$ .

$$S_{n\,,\,0} = u_1 \frac{1-q^n}{1-q} = \frac{1}{2} \frac{1-\frac{1}{2^n}}{1-\frac{1}{2}} = 1 - 2^{-n} .$$

$$S_{n\,,\,1} = \sum_{k=1}^{n} \frac{k}{2^k} \text{ , d'où : } \sum_{k=1}^{n} \frac{k}{2^k} + \sum_{k=1}^{n} \frac{1}{2^k} = \sum_{k=1}^{n} \frac{k+1}{2^k} = 2\sum_{k=1}^{n} \frac{k+1}{2^{k+1}} = 2\sum_{K=2}^{n+1} \frac{K}{2^K}$$

$$S_{n\,,\,1} + S_{n\,,\,0} = 2\left(\sum_{K=1}^{n} \frac{K}{2^K} - \frac{1}{2} + \frac{n+1}{2^{n+1}}\right) ,$$

$$S_{n\,,\,1} + S_{n\,,\,0} = 2S_{n\,,\,1} - 1 + \frac{n+1}{2^n} \Leftrightarrow S_{n\,,\,1} + 1 - 2^{-n} = 2S_{n\,,\,1} - 1 + (n+1)2^{-n} .$$

$$S_{n\,,\,1} = 2 - (n+2)2^{-n} .$$

$$S_{n\,,\,2} = \sum_{k=1}^{n} \frac{k^2}{2^k} \text{ , d'où : } \sum_{k=1}^{n} \frac{k^2}{2^k} + 2\sum_{k=1}^{n} \frac{k}{2^k} + \sum_{k=1}^{n} \frac{1}{2^k} = \sum_{k=1}^{n} \frac{(k+1)^2}{2^k} = \sum_{k=1}^{n} \frac{(k+1)^2}{2^{k+1}} = 2\sum_{K=2}^{n+1} \frac{K^2}{2^K}$$

$$S_{n\,,\,2} + 2S_{n\,,\,1} + S_{n\,,\,0} = 2\left(\sum_{K=1}^{n} \frac{K^2}{2^K} - \frac{1}{2} + \frac{(n+1)^2}{2^{n+1}}\right) ,$$

$$S_{n\,,\,2} + 2S_{n\,,\,1} + S_{n\,,\,0} = 2S_{n\,,\,2} - 1 + (n+1)^2.2^{-n} \Leftrightarrow S_{n\,,\,2} = 4 - 2(n+2)2^{-n} + 1 - 2^{-n} + 1 - (n+1)^2.2^{-n} ,$$

$$S_{n\,,\,2} = 6 - (n^2 + 4n + 6).2^{-n} .$$

On conclue : $\sum_{k=1}^{n} \frac{k^2}{2^k} = 6 - (n^2 + 4n + 6).2^{-n}$ .