Soit l'équation : $\cos 3x - \sin 3x = \sqrt{2}$.

1/ Montrer que $\cos 3x - \sin 3x = \sqrt{2}\cos(3x + \frac{\pi}{4})$.

$\cos(a + b) = \cos a.\cos b - \sin a.\sin b$.

Donc : $\cos(3x + \frac{\pi}{4}) = \cos 3x.\cos\frac{\pi}{4} - \sin 3x.\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}\cos 3x - \frac{\sqrt{2}}{2}\sin 3x = \frac{\sqrt{2}}{2}(\cos 3x - \sin 3x)$,

d'où : $\cos 3x - \sin 3x = \frac{2}{\sqrt{2}}\cos(3x + \frac{\pi}{4}) = \sqrt{2}\cos(3x + \frac{\pi}{4})$.

2/ Résoudre l'équation proposée sur $[\,0\,;2\pi\,[$.

L'équation se ramène à : $\cos(3x + \frac{\pi}{4}) = 0$.

On sait que $\cos A = 0 \Leftrightarrow A = k\pi, \forall k \in \mathbb{Z} \Leftrightarrow A \equiv 0\ [\pi]$.

$3x + \frac{\pi}{4} \equiv 0\ [\pi] \Leftrightarrow 3x \equiv -\frac{\pi}{4}\ [\pi] \Leftrightarrow x \equiv -\frac{\pi}{12}\ [\frac{\pi}{3}] \Leftrightarrow x = -\frac{\pi}{12} + k\frac{\pi}{3}, \forall k \in \mathbb{Z}$.

Si $k = 1$, $x = -\frac{\pi}{12} + \frac{\pi}{3} = \frac{3\pi}{12} = \frac{\pi}{4}$.

Si $k = 2$, $x = -\frac{\pi}{12} + \frac{2\pi}{3} = \frac{7\pi}{12}$.

Si $k = 3$, $x = -\frac{\pi}{12} + \pi = \frac{11\pi}{12}$.

Si $k = 4$, $x = -\frac{\pi}{12} + \frac{4\pi}{3} = \frac{15\pi}{12} = \frac{5\pi}{4}$.

Si $k = 5$, $x = -\frac{\pi}{12} + \frac{5\pi}{3} = \frac{19\pi}{12}$.

Si $k = 6$, $x = -\frac{\pi}{12} + 2\pi = \frac{23\pi}{12}$.

D'où : $S = \{ \frac{\pi}{4} ; \frac{7\pi}{12} ; \frac{11\pi}{12} ; \frac{5\pi}{4} ; \frac{19\pi}{12} ; \frac{23\pi}{12} \}$.