On pose $I_0 = \displaystyle\int_0^{\pi/2} \frac{\sin 2x}{1 + 2\sin x}\, dx$ **et** $I_1 = \displaystyle\int_0^{\pi/2} \frac{\cos x}{1 + 2\sin x}\, dx$ **.**

Soit $I_2 = I_1 + I_0$ **.**

a) Calculer I_2 **.**

$$I_2 = I_1 + I_0 = \int_{0:\pi/2} \frac{\sin 2x + \cos x}{1 + 2\sin x}\, dx = \int_0^{\pi/2} \frac{2\sin x \cos x + \cos x}{1 + 2\sin x}\, dx = \int_0^{\pi/2} \frac{\cos x\,(1 + 2\sin x)}{1 + 2\sin x}\, dx$$

$$I_2 = \int_0^{\pi/2} \cos x\, dx = [\ \sin x\ ]_0^{\pi/2} = \sin \frac{\pi}{2} - \sin 0 = 1.$$

b) Calculer I_1 **.**

Une primitive de $\dfrac{u'}{u}$ est $\ln |\, u \,|$, donc :

$$I_1 = \int_0^{\pi/2} \frac{\cos x}{1 + 2\sin x}\, dx = \frac{1}{2}\int_0^{\pi/2} \frac{2\cos x}{1 + 2\sin x}\, dx = \frac{1}{2}\,[\ \ln |\, 1 + 2\sin x \,|\ ]_0^{\pi/2} = \frac{1}{2} \ln 3\ .$$

c) En déduire I_0 **.**

Par différence, on obtient $I_0 = I_2 - I_1 = 1 - \dfrac{1}{2} \ln 3 \approx 0{,}451.$